CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, Feb./Mar. 2022
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Express $\dfrac{(3+i)(1-3i)}{2+i}$ in the form x + iy. **(06 Marks)**

b. Find the modulus and amplitude of $1-\cos\alpha+i\sin\alpha$. **(05 Marks)**

c. Solve $z^3+1=0$ **(05 Marks)**

OR

2 a. Prove that $\left(\dfrac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^4=\cos8\theta+i\sin8\theta$. **(06 Marks)**

b. Show that if $\vec{a}=i+j+2k$, $\vec{b}=2i-j+k$ then $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$. **(05 Marks)**

c. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are any three non-zero vectors, then prove that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})b-(\vec{a}\cdot\vec{b})\vec{c}$. **(05 Marks)**

Module-2

3 a. Find the n^{th} derivative of $e^{ax}\sin(bx+c)$. **(06 Marks)**

b. If $y=e^{m\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$. **(05 Marks)**

c. Show that the following pair of curves intersect orthogonally $r=a(1+\cos\theta)$, $r=b(1-\cos\theta)$. **(05 Marks)**

OR

4 a. Find the pedal equation to the curve $r=a(1+\sin\theta)$. **(06 Marks)**

b. If $u\cdot(x+y)=x^2+y^2$, then prove that $\left(\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}\right)^2=4\left(1-\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}\right)$. **(05 Marks)**

c. If $u=\dfrac{yz}{x}$, $v=\dfrac{zx}{y}$, $w=\dfrac{xy}{z}$ then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

Module-3

5 a. Obtain the reduction formula for $\int\cos^n x\,dx$, where n being a +ve integer. **(06 Marks)**

b. Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2}dx$ **(05 Marks)**

c. Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx$ **(05 Marks)**

OR

6 a. Evaluate $\int_0^{\pi} x \sin^6 x dx$ **(06 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^6)^{7/2}} dx$ **(05 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$ **(05 Marks)**

Module-4

7 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4$, $z = 3t - 5$, find the components of velocity and acceleration in the direction of the vector $i - 3j + 2k$ at $t = 2$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. **(05 Marks)**

c. Find the constants 'a' and 'b' such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)$ is irrotational. **(05 Marks)**

OR

8 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, +1)$ in the direction $2i - j - 2k$. **(06 Marks)**

b. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$ then prove that $\nabla(r^n) = nr^{n-2}\,\vec{r}$. **(05 Marks)**

c. Show that the vector $\vec{F} = (-2x^2y + yz)i + (xy^2 - xz^2)j + (2xyz - 2x^2y^2)k$ is solenoidal. **(05 Marks)**

Module-5

9 a. Solve $(x^2 - y^2)dx - xydy = 0$. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$. **(05 Marks)**

c. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(05 Marks)**

OR

10 a. Solve $(x + 2y - 3)dx - (2x + y - 3)dy = 0$. **(06 Marks)**

b. Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$. **(05 Marks)**

c. Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. **(05 Marks)**

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